Qualifying Exam Syllabus
Izak Oltman
Date: April 12th, 2021, Time: 9:30am - 12:30pm
Committee: Maciej Zworski (Advisor), Michael Christ, Jim Pitman, Sung-Jin Oh (Exam Chair)

1 Partial Differential Equations (Analysis; Major Topic)

References: *Partial Differential Equations*, Lawrence Evans; *Lecture Notes for Math 222A*, Sung-Jin Oh; *Microlocal Analysis for Differential Operators: An Introduction*, Alain Grigis and Johannes Sjöstrand

1. **Distributions** (Oh 5)

   Basic definitions, basic operations, convolutions, fundamental solutions

2. **Four Important PDE** (Evans 2.2-2.4)

   Laplace's equation: fundamental solution, mean value property, maximum principle, energy methods, Harnack inequality; the heat equation: fundamental solution, regularity/smoothing, maximum principle, energy methods; the wave equation: fundamental solution, finite propagation speed, Huygens' principle, energy methods

3. **Characteristic Equations** (Evans 3.2)

   Derivation, boundary conditions, local solutions

4. **Sobolev Spaces** (Evans 5.2-5.8):

   Basic definitions, approximation, extensions, traces, Gagliardo-Nirenberg-Sobolev inequality, Morrey's inequality, general Sobolev inequality, compactness, Poincaré inequality

5. **Second-order Elliptic Equations** (Evans 6.1-6.5)

   Weak solutions, Lax-Milgram Theorem, existence and uniqueness, elliptic regularity, maximum principles, eigenvalues and eigenfunctions

6. **Second-order Parabolic Equations** (Evans 7.1)

   Definition, existence of weak solutions, regularity, maximum principles

7. **Hyperbolic Equations** (Evans 7.2-7.3)

   Second-order hyperbolic equations: definitions, energy estimates, energy-momentum tensor, finite speed of propagation, regularity; hyperbolic systems of first-order equations: definitions, existence and uniqueness of weak solution

8. **Pseudodifferential Operators** (Grigis and Sjöstrand 1,3,4)

   Oscillatory integrals, basic calculus of pseudodifferential operators, parametrix construction

2 Harmonic Analysis (Analysis; Major Topic)

References: *Euclidian Harmonic Analysis, Notes for Mathematics 258*, Michael Christ

1. **Fourier Inversion, Plancherels's Theorem, and Other Basics:** (Christ 1.1-1.8)

   Definitions of Fourier transforms, convolution, approximate identities, Plancherel's theorem, tempered distributions, Poisson summation formula

2. **Convergence of Fourier Series** (Christ 3.1-3.5, 3.7-3.8)

   Decay of Fourier coefficients, Rademacher functions, Khinchine's inequlaity, uniform and pointwise convergence of Fourier series, almost everywhere divergence (Kolmogorov theorem), L^p norm convergence, almost everywhere convergence, Wiener's Tauberian theorem, Riesz-Thorin Theorem

3. **Hardy-Littlewood Maximal function** (Christ 4.1-4.3, 4.5, 4.7)

   Weak L^p, distribution functions, Hardy-Littlewood maximal function, Marcinkiewicz Interpolation theorem, Calderón-Zygumund decomposition, BMO functions, John-Nirenberg inequality

4. **Singular Integral Operators** (Christ 5.1-5.5)

   Calderón-Zygmund theorem, homogeneous distributions, almost everywhere existence of principal-value integrals, almost everywhere differentiablity, singular integral operators on L^∞

3 Probability Theory (Probability; Minor Topic)

References: *Probability: Theory and Examples*, Rick Durrett

1. **Basic Notions** (Durret 1.1-1.3)

   Measure theory, π–λ theorem, random variables, inequalities, change of variables, notions of convergence of random variables

2. **Law of Large Numbers** (Durret 1.4-1.7)

   Independence, weak law of large numbers, Borel-Cantelli lemmas, strong law of large numbers, Kolmogorov 0–1 law, Kolmogorov maximal inequality

3. **Central Limit Theorem** (Durret 2.2-2.4)

   Convergence in distribution, Helly's selection theorem, characteristic functions, Levy's continuity theorem, central limit theorem, Lindenberg-Feller Theorem